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Voices and values in shaping the subjectivity of pedagogical content knowledge

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Abstract: Pedagogical content knowledge involves subjective decisions on the parts of teachers in making the content comprehensible to learners. This paper is concerned with the formation of this subjectivity and asks: how do (pre-service) teachers come to know and decide upon the best approach to making the content instructional for learners? In answering this question, this study draws upon data obtained from one pre-service mathematics teacher's microteaching and retrospective interviews. The data were examined through the lenses of the Bakhtinian notion of voice. The findings suggest that the pre-service teacher's understanding of pedagogy is formed and transformed, to an important extent, by the voices of others who are not necessarily physically present within the immediate instructional setting and who might be distant in space and time. Assimilation of those voices enacts particular value judgments, which in turn shapes and hence forms the subjectivity of the pre-service teacher's pedagogical practices.

Subjects: Educational Research; Teaching & Learning; Theories of Learning; Teachers & Teacher Education; Mathematics

Keywords: Bakhtin; mathematics teachers; pedagogical content knowledge; subjectivity; value; voice

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PUBLIC INTEREST STATEMENT

Teaching and teachers have always attracted the public attention due to the immediate role that they play for the society. In order for teachers to become effective practitioners, they are expected to make the content understandable to the students. In respect, we, here in this study attempt to understand how teachers come to know the best way of teaching the content. Our understanding is guided and developed by the terms borrowed from Bakhtin, a Russian philosopher and scholar: voice, dialog, and value. On the basis of our findings, we argue that the voices of all those who somehow get into contact with teachers have important effects on the ways in which teaching is performed. The voices also indicate certain values that are reflected in the unfolding teaching practices.

1. Introduction

This paper stems from a project aiming to develop a program for pre-service mathematics teachers (PSMTs) to integrate technology into their teaching (for more on the project, see Özmantar, Akkoç, Bingölbali, Demir, & Ergene, 2010). During the project, the participants' development in terms of Pedagogical Content Knowledge (PCK) was seen as essential, and several consecutive workshops were held (detailed below). After the workshops, it was realized that participating PSMTs displayed different approaches to the delivery of the same content in their plans, in their microteachings, in their partitioning of the lessons, and in their emphases on the aspects of the same concept. They sincerely defended and believed that their approach was suitable for delivering the content in a comprehensible manner. The occurrence of such differences might not be unexpected; of course, PSMTs make personal decisions concerning the delivery of content in ways that they think suitable. Yet, this raises some important questions: How do PSMTs, for instance, come to know what approach is appropriate in making the content comprehensible for learners? Why do they differ dramatically in their approaches despite the fact that they all attend the same workshops, examine the same textbooks, and make use of the same curriculum scripts?

Our attempts to answer such questions focus our attention to the subjectivity of PCK and the sources that form this subjectivity. Our knowledge of these sources, we believe, allows us, teacher educators, to discover certain dynamics of teachers' development with regard to PCK and hence gives an opportunity to achieve desired changes for better instructional practices. In this regard, while explaining how teachers develop their knowledge, Fan (2014) point to several sources: (a) experience as student, (b) pre-service training, (c) in-service training, (d) organized professional activities, (e) informal exchanges with colleagues, (f) reading professional journals and books, (g) own teaching experience and reflection. We, as probably many studying in the field would, agree with Fan (Fan, 2014) that these certainly contribute to the development of teachers' PCK. However, microgenetic analysis of project data and retrospective interviews with PSMTs alerted us to more subtle dynamics (that also lies in the foundation of Fan's reported sources) that form the subjectivity of PCK. In explaining and exploring the subtleties, we found Bakhtinian notion of "voice" (attended to below) rather helpful. Therefore, in this paper we aim to examine and explore the formation of the subjectivity of PCK through the lenses of voice. Although we focus on PSMTs in this paper, we certainly believe that our considerations would have general implications for in-service teachers as well.

In what follows, we shall first begin with a close scrutiny of PCK and attend to the subjective nature of such knowledge. Then we attend to the central Bakhtinian notions of voice and value, which constitute the theoretical framework of this paper. Following this, we shall briefly detail the background of the study and its methodology. The paper ends with a discussion of the issues emerging from the analyses and the implications of the findings.

1.1. Pedagogical content knowledge: Pedagogy, content, knowledge, and subjectivity

PCK was first introduced some 30 years ago by Lee Shulman in his seminal speech at the annual meeting of the American Educational Research Association in 1985. In the following two years, two important journal articles on the construct appeared (Shulman, 1986, 1987). These articles introduced much of the current terminology regarding this notion and are the most often cited articles on the construct. In its simplest form, PCK refers to the knowledge of making content comprehensible to learners and implies the necessary knowledge for the successful teaching of the content at hand. PCK, Shulman (1987, p. 8) writes,

> is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue

Shulman (1987) further argues that the distinguishing feature of an expert teacher is his/her possession of knowledge "to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students (p. 15)." This transformation lies at the core of PCK that Shulman relates to teacher knowledge, which more specifically involves

> the most useful forms of [content] representation ... the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that makes it comprehensible for others ... [PCK] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning. (p. 9)

In Shulman's (1986) view, teacher knowledge is not only classified in different domains (e.g. content and pedagogy) but also comes in different forms: propositional knowledge, case knowledge, and strategic knowledge. Propositions could be research-based principles such as principles of active teaching or experience-based recommendations. These are all in the forms of propositions and are often used in teacher education programs. Norms and values such as providing each student equal opportunity to participate during class discussions are also considered as propositions not because they work in practice but because they are ethically right. Case knowledge, inspired from the case method in law schools, makes a theoretical claim to argue instances of a larger class (Shulman, 1986). Strategic knowledge, on the other hand, "comes into play as the teacher confronts particular situations or problems, whether theoretical, practical, or moral, where principles collide and no simple solution is possible" (Shulman, 1986, pp. 12–13). Strategic knowledge is invoked when principles are in conflict or do not work in practice. It requires professional judgments on the basis of which teachers make instructional decisions.

The notion of PCK was welcomed by many educators following its introduction; since then, it has gained tremendous popularity in the circles of teacher educators. Within the last 30 years or so, there have appeared numerous research articles, dissertations, and conference papers focusing on this notion. The notion has also become an indispensable part of course syllabi for both in- and pre-service teacher education around the world. One might speculate that one reason for the notion's popularity is that it provides researchers, practitioners and policy-makers alike with a useful tool to talk about the practice of "good" teaching (and a "good" teacher, for that matter). The idea behind PCK sounds "simple," as if "representing common sense" (Bullough, 2001). It is perhaps this ostensible simplicity that has led many research articles to make use of the notion through in-passing citations of Shulman as the author (see Segall, 2004) without really scrutinizing the notion itself, which is hence often taken for granted.

When scrutinized carefully, it can be realized that PCK is mainly concerned with making content comprehensible for learners by virtue of the most "powerful" formulation of the content and most "useful" forms of representation. Achievement of this also depends on the forms of knowledge that teachers develop and employ. It should be noted that all forms of teacher knowledge (propositional, case and strategic knowledge) that Shulman (1986) had in mind are subjective in nature. This is particularly because norms and values in the forms of propositions are moral, ideological or philosophical commitments on the basis of which (instructional) decisions are made and professional judgments are performed. Teachers, for instance, make judgments as to, and hence have knowledge of, things that make learning of specific content easy or difficult. However, "useful" and "powerful" are value-laden expressions and demand teachers make subjective decisions upon devising ways to represent and formulate the content in a comprehensible manner. Further to this, judgments on the "things" that facilitate learning a particular topic are also rather personal and depend once again on the subjective decisions of teachers.

As is apparent from the considerations given so far PCK is subjective in nature. However, subjectivity of PCK should be considered with regard to teacher's enactment of pedagogical knowledge in his/

her practices. That is, teachers make particular decisions and perform certain actions in order to intentionally influence the production of an understanding on the part of a particular group of learners. Their actions and decisions in making the content meaningful to the students, hence, can be considered as a particular enactment of PCK. Hence, when we use the term "subjectivity of PCK" throughout the paper we are in fact referring to PCK as enacted by a teacher.

Considering this subjectivity, this paper asks: How do (pre-service) teachers come to know the most useful and powerful representation and/or formulation of the content in ways that make it comprehensible to learners?

An attempt to answer this question might consider the notion of belief. Teachers' beliefs have been a focus of attention in teacher education research, especially in the last three decades because belief has been considered to be affecting teachers' practices (Kagan, 1990; Lui & Bonner, 2016; Ng, Nicholas, & Williams, 2010; Stipek, Givvin, Salmon, & MacGyvers, 2001; Uibu, Salo, Ugaste, & Rasku-Puttonen, 2017). Gess-Newsome (1999) claims that "beliefs can be seen as information filters and impacting how knowledge is used, organized and retrieved … and are also powerful predictors of behavior" (p. 55). Likewise, many studies discuss beliefs in line with PCK to explore teachers' practices (Gess-Newsome, 1999, 2015; Magnusson, Krajcik, & Borko, 1999). On the other hand, research on teachers' beliefs often focuses on consistencies (Stipek et al., 2001) or inconsistencies between belief and practice (Brown & Melear, 2006; Raymond, 1997; Tsai, 2002). Findings of these studies demonstrating different beliefs and practices have been considered as "inconsistencies" without acknowledging the role of values, contextual or cultural factors.

Therefore, we direct our attention to the notion itself in search of an answer to the above-posed question. As noted above, PCK was originally formulated as a blend of content and pedagogy. Thus, an explanation of subjectivity, in our view, needs to consider both the content and pedagogical dimensions together: Could the mentioned subjectivity be related to the content knowledge of teachers; and if so, how? What is the role of pedagogy for the subjectivity of PCK? Below, we attend to these questions in separate sections.

1.2. Content knowledge as a source of subjectivity?

It is useful to note here that content knowledge, generally speaking, refers to the knowledge one has about a subject such as mathematics, including its epistemology and structures (see Graeber, 1999). More specifically content knowledge involves the comprehension of key facts, concepts, principles, and explanatory frameworks in the discipline, as well as the apprehension of what counts as evidence and proof within the given discipline (Shulman & Grossman, 1988). There have been many studies focusing on the role of teachers' content knowledge. Some considered content knowledge in relation to PCK (e.g. Galuzzo, Leali, & Loomis, 2000; Hutchison, 1997; Manouchehri, 1997); others did so without an explicit reference to the notion, though these studies certainly have clear implications for teachers' PCK (e.g. Carlson, 1988). Regardless of whether studies related the content knowledge to PCK, they nevertheless provide substantial evidence that content knowledge impacts, for instance, teachers' questioning techniques (Carlson, 1988), achievement of productive discourses in mathematics classrooms (Manouchehri, 1997), decisions on curriculum (Leinhardt & Smith, 1985), quality of feedback (Galuzzo et al., 2000), and explanations (Hutchison, 1997), as well as privileging the form of understanding promoted through instruction, e.g. relational or instrumental (Kinach, 2002). These studies make it sufficiently clear that one of the sources that shape the subjective decisions of teachers is related to the depth of content knowledge, which surely makes differences in their practices.

So far so good. Depth of subject-matter knowledge is one source of subjectivity for PCK; however, the picture is far from being complete for the obvious reason that even those who have strong subject-matter knowledge often approach the delivery of content differently. This was the case in this study. The participants of the project were given a diagnostic test on the mathematical content pursued during the workshops (detailed later) to see whether they had the necessary conceptual

understanding. The test was applied twice: before and after the workshops. It is true that some of the pre-service teachers were lacking in conceptual understanding before the course but others' responses demonstrated an explicit conceptual understanding of the meanings and definitions for the topic in focus. Yet, their practices differed considerably. Hence, subject-matter knowledge alone is not sufficient to explain the subjectivity of PCK. We turn our attention to the second question posed above: what does pedagogy have to offer in explaining this subjectivity?

1.3. Pedagogy as a source of subjectivity?

Examination of the literature related to the subjectivity of PCK reveals that neither Shulman and his colleagues nor subsequent research engagements have paid sufficient attention to pedagogy. There appears to be a tendency among the researchers to employ PCK for teacher preparation and determination of the quality of their instructional practices (Segall, 2004). Many dealing with PCK treated pedagogy as if it were self-evident. This at times led to oversimplified considerations of pedagogy, such as teachers' knowledge of teaching and methods of instruction in classrooms (Segall, 2004). Thus viewed, pedagogy becomes imprisoned within walls of classrooms; disassociated from the implications of discourse, power, and value; and ignored as producing a culture and being produced culturally (Daniels, 2001; Gudmundsdottir, 1990). This view also reduces teachers' pedagogical practices to decoding of the text (Giroux, 2000). Therefore, research literature revolving around PCK does not much extend our understanding of the relationship between the pedagogy and subjectivity of PCK.

In exploring this relationship, the literature from critical pedagogy provides promising insights. In this area of research, pedagogy in its simplest form is concerned with the conditions under which knowledge (or meaning) is produced (Lusted, 1986). In effect, pedagogy aims to regulate and organize the practice of knowledge production. It is through pedagogy, writes Simon (1992, p. 59), that "meanings are absorbed, recognized, understood, accepted, confirmed, and connected as well as challenged, distorted, taken further, or dismissed." Surely these acts are all intentional and depend on teachers' perspectives on knowledge production and hence are subjective in nature.

Davies (1994) also sees pedagogy with regard to knowledge production, but he relates this to individuals' understandings of educational aims:

> pedagogy involves a vision (theory, set of beliefs) about society, human nature, knowledge and production, in relation to educational ends, with terms and rules inserted as to the practical and mundane means of their realisation. (p. 26)

Here, Davies, in our view, clearly articulates the subjective nature of pedagogy in relation to practice. That is, pedagogic practice depends on (and is shaped by) the beliefs one holds, on teacher judgments about what characterizes, motivates and achieves the learning; on interpretations of how the resulting learning (or produced knowledge) relates to educational ends; and on execution of actions for getting to these ends.

The subjectivity of pedagogical practices is viewed in relation to institutional contexts by Giroux and Simon (1988, p. 12); they note that pedagogy

> organize[s] a view of how a teacher's work within an institutional context, specifies a particular version of what knowledge is of most worth, in what direction we should desire, what it means to know something, and how we might construct representations of ourselves, others, and our physical and social environment.

It is crucial to note that Giroux and Simon's consideration of institutional contexts well beyond schools. They see pedagogy developing at any site where cultural production and reproduction takes place. The kinds of sites that the authors have in mind vary greatly, ranging from film-making to engineering and advertising. These sites engage in forms of cultural work just as do schools. The authors argue that social relations, practices, experiences, and value judgments gained within and

among cultural sites can exercise an influence upon the development of teachers' pedagogy and hence their work in schools.

As the considerations hitherto suggest, pedagogy is concerned with the conditions under which knowledge is produced. These conditions are, to a great extent, related to teachers' views, decisions, beliefs, and value judgments on a wide range of issues, all of which are personal and hence subjective. One can safely conclude that pedagogy plays a vital role in explaining the subjectivity of PCK. Although research on pedagogy has clear implications regarding the subjectivity of PCK, it does not provide a framework to explain the development of this subjectivity based on empirical data. For this issue, we found Bakhtin, who focused extensively on the development of worldviews, value judgments, and ideas, helpful. In this respect, two Bakhtinian notions are particularly relevant to the focus of this paper: voice and value, which constitute the theoretical framework of this paper and are attended to next.

2. Theoretical framework: Bakhtinian notion of voice and value

One of Bakhtin's main interests was the creative aspect of human language. To Bakhtin, this creativity can be found in the plurality of human experience and dialogicality of human speech. Bakhtin considers dialog as a continuous interaction between the meanings that affect others (Emerson & Holquist, 1981). The dialog in his view goes well beyond face-to-face verbal interaction and involves verbal communication of any type whatsoever. Thus viewed, even a book (i.e. "verbal performance in print") is in essence dialogic as "it responds to something, objects to something, affirms something, anticipates possible responses and objections, seek[s] support and so on" (Voloshinov, 1973, p. 95).

In Bakhtin's approach to the analysis of dialogicality in verbal communication, the notion of voice plays a central role. To him, voice is the speaking consciousness or personality which always has a will or desire behind it (Emerson & Holquist, 1981, p. 434). By coming into contact with other voices, the will or desire reflects itself in the production of discourse through utterances by assessing the words of others, judging the veracity, arguing, objecting, and so on. By voice, Bakhtin stresses broader issues of the speaking personality's conceptual horizon, points of view, perspective, and intention in producing particular utterances (Wertsch, 1991). Utterances are seen as "the real unit of speech communication" (Bakhtin, 1986, p. 71), and their existence depends upon the co-existence of voices; that is, at least two voices (speaker and addressee) are required for utterances to come into being. Any utterance, whether spoken or written, is always expressed from a point of view (Wertsch, 1991) and always directed to at least one addressee. The voice, whether it be speaker or addressee, varies from being an immediate participant of the dialog to an "indefinite, unconcretized *other*" (Bakhtin, 1986). Bakhtin (1986) argues that responsivity creates a connection between the speaker and addressee in producing particular utterances. The speaker produces utterances in anticipation of a response and those utterances themselves are responses to some previous ones either from within the immediate dialog or from a previous one. The addressee takes an active and responsive attitude toward the speaker's utterance: "sooner or later what is heard and actively understood will find its response in the subsequent speech or behavior of the listener" (Bakhtin, 1986, p. 69).

Bakhtin argues that an individual's discourse is constantly interwoven with the speech of others and this in fact constitutes the source for the development of personal voice:

> The unique speech experience of each individual is shaped and developed in continuous and constant interaction with others' individual utterances. This experience can be characterized to some degree as the process of assimilation–more or less creative–of others' words (and not the words of language). Our speech, that is, all our utterances (including creative works) is filled with other's words, varying degrees of otherness or varying degrees of "our-own-ness," varying degrees of awareness and detachment. These words of others carry with them their own expression, their own evaluative tone, which we assimilate, rework and accentuate (1986, p. 89).

As this quotation indicates, Bakhtin assigns a focal position to the words and voices of others for the development of personal voice. That is, while producing particular utterances, voices either take on and reproduce others' words as if they were their own (our-own-ness) or through reported speech (Voloshinov, 1973). The production of unique utterances in virtue of other's voices involves a particular dialogicality that Bakhtin termed ventriloquation, which is the process whereby "one voice speaks through another voice in a social language" (Wertsch, 1991, p. 59). However, invoking the voice of others does not occur on a random basis. Individuals are selective while assimilating the words and voices of others. This selective assimilation involves, to some extent, incorporation of the others' conceptual horizon, point of view, and worldview into one's own perspective and plays a central role in the maturation of an individual's self-consciousness. Thus, any voice with its own specific utterances inevitably conveys commitments and enacts particular value judgments as the words of others carry with them their own expression and own evaluative tone.

Based on the considerations of pedagogy and voice, we believe that the subjectivity of PCK is closely related to teachers' pedagogy as manifested through their practice and discourse. We will examine the subjectivity of PCK through the lens of the Bakhtinian notion of voice. In pre-service teacher education research, studies on the interaction between pre-service teachers and university mentors implicitly points out the effects of the teacher educator's voice on shaping pre-service teachers' instructional beliefs, professional identities, and teaching practices (Borko & Mayfield, 1995; Peressini, Borko, Romagnano, Knuth, & Willis, 2004; Tsui & Law, 2007). For example, Postlethwaite and Haggarty (2012) investigated the ways in which pre-service mathematics teachers dealt with the differences between ideas that were valued in university and the school contexts. Likewise, Shulman (2015) emphasizes the role of non-cognitive attributes such as context and culture in the formation of PCK. According to him, PCK must "be mindfully situated in the disciplinary, cultural, personal, and social settings in which it occurs" (p. 9). Our argument is hence that teachers' understanding of pedagogy develops through dialog with the others' voices; assimilation of those voices enacts particular value judgments, which in turn shape teachers' practices. Voices and values therefore determine the subjectivity of teachers' PCK. We will exemplify this argument based on empirical data in the rest of the paper.

3. Background of the study and methodology

This paper stemmed from a research project that aimed to develop a program for pre-service mathematics teachers to integrate digital technologies into their teaching (for more on the project, see Özmantar et al., 2010). For this purpose, the notion of technological Pedagogical Content Knowledge (Mıshra & Koehler, 2006) was adopted as a theoretical framework resting upon the notion of PCK. Hence, as part of this project, an initial step was to design a course with the aim of participants' development of PCK. In designing the course and developing the content, the relevant literature was reviewed. There were many efforts to operationalize PCK for the purpose of research and practice (e.g. Kinach, 2002). In this connection, several studies differentiated certain components in relation to PCK. Two such components were extracted directly from Shulman's initial studies: student (mis) understandings as well as teaching strategies and representations. Later exploration of PCK led to development of different models with additional components (e.g. knowledge of assessment and of curriculum) (for an overview, see Park & Oliver, 2008). In designing the course, Magnusson et al.'s (1999) five components of PCK were employed: knowledge of multiple representations, of student difficulties, of instructional strategies for teaching, of assessment, and of mathematics curricula. The course designed for the development of pre-service teachers' PCK involved all of these components.

At the first stage of the course design, objectives regarding each component were formulated. At the second phase, the content for each component was specified. Relevant literature and curricular materials were used to develop the content for each component. Five components of PCK were discussed in the context of two mathematical concepts, function and derivative (e.g. multiple representations of function and derivative concepts). At the third stage, workshop method was embraced as the method of delivery (organization of learning experiences). Workshops included hands-on activities such as lesson planning, use of mathematics software tools, micro-teaching, self- and

peer-assessment of lessons. The second author of this paper conducted the first set of workshops. An important aspect of the course design is that pre-service teachers, in groups of four and five, planned and conducted their own workshops. Each group focused on a different mathematical concept such as limit, continuity, radian, probability and integral. The aim of the second set of workshops was to help pre-service teachers develop a PCK perspective with its five components.

3.1. Participants

The participants of the project were a group of forty pre-service teachers who studied mathematics for three-and-half years. They enrolled in a teacher-preparation program, graduates of which teach mathematics at secondary level. During the preparation period, they took mathematics method courses (e.g. Technology-Aided Mathematics Teaching), as well as other teacher-education modules (e.g. Educational Psychology). The data collection for this study took place during the module of "The Methods for Teaching Mathematics." In this module, the participants initially attended separate workshops on each of the components of PCK cited above. Following this, application of these components was exemplified based on a particular mathematical concept, namely, derivative. During this process, the participants were asked to read certain texts on each of the components (e.g. Özmantar, Bingölbali, & Akkoç, 2008). They also examined curriculum scripts, prepared lesson plans and detailed teaching notes for the introduction of derivative, and performed micro-teaching lessons where they taught the notion of derivative to their peers. This study focuses on a single pre-service teacher (Banu, not her real name) since investigation of subjectivity of PCK requires an in-depth exploration.

3.2. Data-collection tools

Several data-collection tools were employed for this study: lesson plans, teaching notes, micro-teaching video records, interviews, observation forms, and a test on derivative. Participants took two calculus courses in their first years; therefore they have already met the derivative concept. Still, before the workshops started, it was important to gain insights into the participants' conceptualizations and interpretations of the derivative concept. Hence, a test on this topic was developed and applied to the participants. The test included questions to investigate understanding of different aspects of derivative (Bingölbali, 2015): derivative as instantaneous rate of change, derivative as a slope and derivative as a limit. Participants were asked to give their personal definitions of derivative, to find instantaneous velocity in the context of physics when the function is represented algebraically and graphically, to derivative at a point when the function was given as table of values.

All the participants also prepared lesson plans on two occasions: before and after the workshops. The aim was to observe the changes in their lesson plans following the workshops. The participants were also required to get ready detailed teaching notes to use during their micro-teaching lessons. During the module, groups of pre-service teachers were, respectively, assigned several topics such as functions, limit, continuity, derivative, integral, probability and trigonometry. They were asked to prepare lesson plans targeting at upper-secondary students. During the micro-teaching lessons, classmates of the pre-service teacher were advised to act like students who learnt about the content for the first time. At the end of the lesson, they evaluated the lesson using an observation form, which focused on the components of PCK.

They also filled in observation forms with open-ended items on each of the components of PCK to gain insights into their development. The form required participants to evaluate micro-teaching in terms multiple representations, student difficulties, instructional methods and strategies, assessment methods and evaluation.

Data were also collected through a video recording of micro-teaching sessions that took place just after the completion of the PCK workshops. Finally, participants were interviewed about their lesson plans as well as their preparation and conduct of micro-teaching lessons. The interview on their lesson preparation was based on the five components of PCK: knowledge of curriculum and curricular materials (how they make use of them in their preparation), knowledge of multiple representation

(which representations used and why), knowledge of students' difficulties (how to address difficulties in lesson preparation), knowledge of instructional methods and strategies (rationale for choosing a particular method), and knowledge of assessment (plans for assessment). In the second interview, participants were asked to reflect on their lessons based on five components of PCK as described above.

3.3. Data collection and analysis procedures

We mainly present data obtained from Banu's micro-teaching lesson, interview on her lesson preparation, retrospective interviews on her practices, her teaching notes, and her lesson plans. Before presenting data, however, it is necessary to briefly explain the data collection and analysis procedures, which were intertwined at certain points.

To gain insights into participants' teaching approaches, records of micro-teaching videos, including Banu's, with lesson plans and teaching notes were initially examined. This examination focused mainly upon the pre-service teachers' pedagogical practices, which are considered to be intentional actions and decisions that try to influence the production of meaning (Giroux & Simon, 1988). Hence, attention was focused on pre-service teachers' actions and decisions during their teachings, and on the partitioning and structure of the lessons.

The videos of micro-teaching lessons were analyzed separately by two researchers (authors of this paper) using a negotiated framework. Based on this framework, the videos were fragmented into segments which represent an independent task in a lesson. Each researcher wrote a descriptive summary for each segment making references to different aspects of derivative. Researchers then compare their analysis based on the fragmentation and the descriptions of each segment. Pre-service teachers' approaches to the delivery of the content (i.e. introduction of derivative) were critically evaluated. Based on this examination and evaluation, certain issues were determined. These issues embodied differences in the way the topic was delivered in terms of the emphasis put on different aspects of the same topic, the ordering of those aspects, and the departure point of the lesson. For example, for the introduction of derivative, one pre-service teacher began by directly presenting the definition of derivative, while another posed a problem about velocity/motion and only after the solution of the problem gave the formal (i.e. algebraic) definition.

To gain insights into how pre-service teachers decide which approach is best for introducing the derivative concept and the reasons for these differences, we particularly focused on Banu's micro-teaching records. Based on the examination of her micro-teaching lesson, she was interviewed retrospectively. During the interviewing, Banu watched the record of her micro-teaching together with the interviewer in a one-to-one situation and, when needed, the video was paused at those moments determined beforehand considering the segments determined by the researchers. Questions were directed to Banu with the aim of uncovering the reasons for and rationales behind her pedagogical actions and decisions.

In what follows, we will focus on Banu's micro-teaching and also present data obtained from retrospective interviews with her. We analyze the data with reference to her pedagogical practice.

4. Data analysis

Data analysis is presented in two sections. The first provides an overview of Banu's micro-teaching. The second part provides excerpts from retrospective interviews in relation to Banu's teaching practice.

Banu began her micro-teaching lesson by noting that derivative was used in different domains of real life, such as engineering and economics, and talked about several examples (e.g. calculating the maximum profit in economics). She then provided physical and geometrical interpretations of derivative based on a problem. While explaining their physical interpretation, she made use of the distance equation (velocity multiplied by time) and computed the instantaneous velocity at a particular

point by evaluating the average velocity over various intervals in the neighborhood of that point. She then mentioned the notion of limit, which she used to find the geometrical interpretation of a derivative while calculating the slope of a tangent line at a point on a curve. Following this, she attempted to connect physical and geometrical interpretations, which were then used to provide the formal definition of a derivative. She exemplified and explained how to use the formal definition to calculate the slope of a tangent line at a particular point with a specific function ($y = x^2$, at $x = 2$). Later she noted that application of the formal definition of the derivative was a long process involving tedious work and that there were shorter ways to find derivatives with certain rules. She ended her lesson by assigning homework for students to search for real-life situations in which derivative could be used.

For the rest of data analysis section, we provide excerpts from two interviews with Banu: one was conducted after her micro-teaching to find out about the details of her preparation and hear her reflect on her practice during teaching. The second was conducted retrospectively after the analysis of her micro-teaching on the subject of her pedagogical practice. We present selective excerpts with reference to her teaching practice insofar as they are related to the focus of the paper. While presenting the excerpts, we insert interpretive comments that are not necessarily comprehensive or definitive (Wells, 1999). Our purpose with these comments is to offer possibilities of interpretation, rather than to establish rigid meanings.

Although several PSMTs started their teaching with a formal (limit) definition of derivative, Banu preferred to start her teaching by explaining where the concept of derivative was used in real life and provided several examples. When asked about the reasons, she replied:

> Students always say, “we learn these but where we use them, what use they have?” and generally this is the breaking point from the topic taught. I thought explaining where it is used is important to motivate students. I have friends … they’re engineering students and often say “we learnt derivative but only now (when they apply it in various contexts) we understood why it is important and useful.” To stress this I explained where it is used.

Banu’s pedagogical decision to provide examples for the use of derivative in different fields is saturated with the voices of those previously met students who question the benefit of learning derivative and who study in engineering departments. These voices were so influential on her practice that she felt the need to talk about examples of the daily use of derivative. In fact, this was a kind of propositional knowledge in Shulman’s (1986) terms and is commonly used in teacher education programs and the Turkish national curriculum of mathematics. However, Banu’s pedagogical decision is also an indication of her strategic knowledge. Her introduction to the topic through real-life examples is based on reflections on her experiences with students. We can see value judgments as Banu allows others’ voices to enter her discourse. She is in fact selectively assimilating, that is valuing those voices with an impact on her instructional decisions and actions.

The examples Banu provided for the use of derivative in real life were related to economics and engineering. She was asked where she learned about these:

> I knew derivative was used for the calculation of profits in Economy; we came across such problems during university in calculus lessons. Apart from this, I know derivative is used in engineering because both my father and brother are engineers. My brother is a mechanical engineer and he often employs derivative for the error calculations for sketches in Autocad and asked me about it.

It is interesting to see here that Banu’s pedagogical decision to provide examples for real-life usage of derivative is, to a considerable extent, informed by her interaction with her father and brother whose voices ventriloquate through Banu’s speech. She apparently favored their voices, accentuated and put so much value that designed her teaching accordingly. The effect becomes particularly evident when we realize that Banu ended her teaching by assigning homework for students to find a field in real life where derivative were used. This was particularly different from her peers, many of

whom ended their teaching by giving worksheets with problems related to derivative. When she was asked about the source of her belief in the necessity of this practice, she responded:

> We were not taught where derivative was used. I was scared of making mistakes and shaking while doing problems on derivative … with all these symbols and abstract things … signs. I'd want students to search and find concrete examples of its use … it shouldn't remain as an abstract thing … I aimed to concretize things in student mind, freeing students from the signs and symbols and getting them into real life and learn things better and meaningful.

We can see Banu's recourse to her own experience as a student when she said she was not taught derivative in a concrete manner. This time her reflection is based on her own learning experience. She is apparently against teaching derivative in an "abstract" way through signs and symbols. This reflects her view of what element is important in teaching mathematics: concretizing. This element indeed specifies (and in fact prescribes) a particular direction that one should desire for knowledge production: "freeing students from the symbols and getting them into real life."

Banu's approach to introducing derivative was evidently different from many of her peers. Instead of directly presenting the definition of derivative, she made use of physical and geometrical interpretations and employed a problem to link these two together. Only then, after delving into physical and geometrical interpretations, did she present the formal (algebraic) limit definition of a derivative. When asked about the reason for introducing derivative in this way, she replied:

> You introduce derivative for the first time … you get students sensing the applicability of their knowledge from analytic geometry, from physics. I mean interpretation of derivative stems from the idea of slope from analytic geometry … they already know very well from physics: distance equals velocity multiplied by time, which can be used to teach derivative. I tried to build on their old, existing knowledge. Step by step. Wanted them to make connections with their old knowledge and thus construct new knowledge … Teach them to use what is known.

It is apparent that Banu's pedagogical approach was shaped by her perspective on knowledge production. She tries to make use of what students know while constructing new knowledge structures. Important here is the question of how she has come to adopt such an approach. Her answer was:

> I initially examined textbooks. I realized that these were treated as if they were completely unrelated (to each other and to the definition of derivative) … their approach is confusing … It's the same in the schools where I attended to observe teachers. They give rules of derivative before explaining geometrical and physical interpretation … like abstract mathematics with proofs … I can say I learnt derivate at the university so I looked into course notes. Our university lecturer introduced derivative through geometrical and physical interpretations. So, I also thought of obtaining the definition [of derivative] from these.

Banu's pedagogical approach to introducing derivative through physical and geometrical interpretations was shaped through her interaction with the voices of many, including authors of textbooks, the teachers she observed during her school placement, and her university lecturers. She clearly recognized the approaches of the textbooks and teachers that she observed and university lecturers from whom she learned derivative. However, while challenging and in fact dismissing the approaches of textbooks and teachers, she was accepting, confirming and even adopting the approach of her university lecturers. In a sense, she was assimilating the words and voices of her university lecturers, which constituted a source for her to develop her pedagogical approach. This assimilation reflects certain value judgments that fundamentally shape her pedagogical practice. Apparently, Banu valued teaching the derivative by connecting physical and geometrical interpretations that, to her, were treated as if unrelated in textbooks.

There was an interesting incident of Banu's practice that further reflects the effect of others' voices and value judgments. After she presented the formal definition of a derivative, Banu solved an

example (finding the derivative of x^2 at $x = 2$) to show the application of the formal definition in calculating the derivative at a point. After the example, she told the class that "later we won't be tussling all over these, there is a short way." In fact, she was the only one who described the use of derivative definition as "a tussle." When asked why she felt the need to mention the shorter way:

> In a private tutoring (of a student), while defining the derivative, the student asked if there wasn't a short way of finding derivative. So (during the micro-teaching) I wanted to tell that students wouldn't need to tussle with all these (application of formal definition to find derivative of a function at a point), there are short ways to find derivative.

None of Banu's peers during her micro-teaching lesson asked if there was a short way to find the derivative; nevertheless, she mentioned this. She was in a sense responding to the voice of a student that she tutored in the past. This response enacts certain value judgments. She viewed finding derivatives through the application of the formal definition as a "tussle" and hence, by implication, valued the "short way" (i.e. differentiation at a point using the rules of differentiation). In fact, the voice of a single student is not the only issue here. The expectation of this single student is also the voice of an education system shaped by high-stake examinations, and Banu's approach is shaped by these voices. There is a conflict between the physical and geometric interpretation approach and the "short way" approach. Resolving this conflict depends on strategic knowledge which could be developed through reflections on her practice in the classroom when she enters the profession.

Banu's selection of examples was also a focus of the retrospective interview. When asked about how she decided upon the selection of examples, she noted:

> I was concerned to find appropriate examples. The examples should be ordered so that they get increasingly difficult. I search in my university handouts. I selected the simple ones and had difficult ones as well. In micro-teaching, I started with the simplest examples, which were doable immediately following the definition. We need to move from the simple to the complex in an effective instruction.

Banu reflects her understanding of human development, of what makes learning easy or difficult, and of the elements that effective teaching involves while choosing examples: instruction should provide a transition from the simple to the complex. But beyond this, there was also the factor of visualization that impacted her decision while both selecting the examples and teaching the notion:

> While choosing examples I paid attention that they should allow the visualization. Actually I was careful to let students visualize the things I explained. Because verbal expressions are difficult to visualize. It is still firmly in my mind what my lecturers [at university] did while getting the graphs sketched. I used graphs and tables in my teaching and solution of examples because numbers stand still in the air.

As far as Banu's excerpts provided hitherto make clear, a successful teaching of derivative (and hence, by implication, mathematics), to her, involves the elements of visualization, concretizing, moving from the simple to the complex, constructing definitions, and making connections between the old (constructed) and new (to be constructed) knowledge. Although these elements seem to reflect a vision on the production of mathematical knowledge and on the way in which effective instruction can be performed, Banu's approach might also have been saturated with the voices of the teacher education program she participated. Principles, whether research or experience-based, such as "moving from simple to complex examples" or "visualization promotes understanding" are all promoted by the teacher education program she attended. Therefore, Banu's propositional knowledge which includes these principles has been shaped by the voices of the researchers which designed and held the workshops.

5. Discussion

Analysis of Banu's retrospective interviews along with her micro-teaching records makes it clear that she was convinced of the usefulness of her approach in delivering the content (i.e. introduction of derivative). She also believed that her approach was better than at least that of the textbooks she examined and of teachers that she observed during her school placement. She stated that the structure of her teaching and presenting derivative through physical and geometrical interpretations was suitable and beneficial for learners to comprehend the content, and indeed defended this position.

We do not make judgments on the effectiveness and/or appropriateness of her or textbooks' approach here in this paper. Yet, the way she approached teaching derivative indicates her understanding of how to make this particular content comprehensible to learners and hence embodies Banu's PCK of derivative. This understanding shaped her practice and rested upon Banu's value judgments regarding what elements make successful teaching of mathematics possible: presenting the content to enable students to visualize; making connections between what is known and what is to be known; concretizing the content by means of, for instance, real-life examples; organizing the content and examples starting from the simple and progressing to the complex; constructing definitions together with students, rather than supplying them directly. These elements seem to point to Banu's vision of knowledge and production, human development, learners, and characterization of learning. This vision is at the heart of her understanding of pedagogy for teaching mathematics. It is by virtue of this vision that Banu blended the content (derivative) and pedagogy and hence constituted the basis of her PCK with terms and rules inserted into her pedagogical practice (Davies, 1994, p. 26) in formulating and presenting the content. That is, she used tables and graphs to enable visualization, making use of distance and velocity from physics and slope from analytic geometry (what is known) to reach the definition of derivative (what is to be known), such that students could connect their existing knowledge structures with the new ones; presenting examples of real-life usage for derivative to concretize the concept; demonstrating the application of the formal definition of derivative starting with simple examples.

Analysis of retrospective interviews on Banu's micro-teaching suggests that her discourse imbued with others' words and voices which reflect certain values that shape her pedagogical practice. Voices of others who were not physically present within the immediate context of Banu's micro-teaching ventriloquated through her speech: those of friends/students from engineering departments, her brother as an engineer, past teachers, university lecturers, authors of textbooks, and designers of TPCK workshops. Banu's observed pedagogical practices and discourses, as Bakhtin stresses, are shaped and developed in constant interaction with the words and voices of others. These voices sometimes reveal themselves in the forms of propositions (research or experienced-based principles) as described by Shulman (1986) e.g. "visualization promotes understanding."

However, others' voices did not readily enter into Banu's own world. She assimilated them selectively; while she was accepting and accentuating of some voices, she was challenging and even dismissive of others in the course of her practice. In either case, her value judgments were observable. For instance, she recognized the approach adopted by textbooks and her past teachers, but she challenged the voices of the textbook writers (as they treated physical and geometrical interpretations as unrelated) and of teachers (as they tended to teach derivative abstractly with "signs and symbols"). Apparently, she "de-valued" such prescriptions. However, she assimilated the words and voices of her university lecturers (and therefore adopted their approach), of the students questioning the usefulness of learning derivative (and hence gave examples of real-life use and assigned homework for that matter), and of asking for the "shorter" way to calculate derivatives (hence she assured the class that there would be short ways). The effect of those voices on Banu's pedagogical practices was all too apparent. Thus, selective assimilation of the voices impacts deeply on the formation of the subjectivity of PCK, which is manifested by virtue of the actions and decisions of teachers in selecting the most useful representation and formulation of the content, as well as their conduct of pedagogical practices. This implies strategic knowledge which requires teachers' "reflection leading to self-knowledge" (Shulman, 1986, p. 13). Based on these observations, we contend

that others' voices (either assimilated or challenged or even rejected) lead to the development of a unique understanding of pedagogy for individuals.

An important implication of the involvement of others' voices in Banu's practice is that her understanding of pedagogy develops under the influence of different cultural sites, as contended by Giroux and Simon (1988). Her practice was shaped by voices from such cultural sites as engineering departments, her family, her university, schools, textbook production and TPCK workshops. Banu's social relations, practices and experiences gained in different cultural sites influenced the development of her understanding of pedagogy. This suggests that development of pedagogy transcends well beyond the walls of classrooms and is not limited to teachers' experiences in schools. On this basis, we argue that the voices culled from a wide range of textual sources, historical periods, and social experiences (see Gardiner, 1992) gained in different cultural sites contribute greatly to the development of teachers' pedagogy, as well as their pedagogical practices with regard to PCK.

Having discussed the influence of others' voices upon the development of teachers' pedagogical practices, we must note a necessary consequence of this position that development of teachers' pedagogy is a fundamentally historical process and not a singular event. In other words, development of teachers' understanding of pedagogy is a continuous dialogic process with the others and construction of a complete understanding is not achievable, and is always unfinalized. This insight is evident in Bakhtin's writings on the self; in this respect, Bakhtin biographers, Clarke and Holquist (1984, p. 65) point out that:

> the self, conceived by Bakhtin, is not a presence wherein is lodged the ultimate guarantor of unified meaning. The Bakhtinian self is never whole, since it can only exist dialogically. It is not a substance or essence in its own right but exists in a tensile relationship with all that is other and, most importantly, with other selves.

The implication of this quotation for the development of teachers' pedagogy is that as long as others remain to exist, teachers are formed and transformed continually and thus there will always be changes in their understandings (this change does not have to be marked with positive terms though). Therefore, the subjectivity of PCK is shaped by the words and voices of others and cannot be considered as a singular and to-be-completed process; it can be best described as an ongoing but never finalized. The subjectivity of PCK is constituted thus through the operation of a dense but at times conflicting network of others' utterances/discourses, cultural and social practices in and among different cultural sites. The multiplicity of such interactions and experiences is one important factor contributing to the formation of the subjectivity of PCK.

The voices assimilated and hence entered into one's own discourse carry with them certain value judgments. This is because every voice speaks from a particular point of view with a specific conceptual horizon and within the specific world of the individuals. During dialogical relationships, Bakhtin (1981) stresses, these various points of view, conceptual horizons, and specific worlds of individuals come into contact with one another. He also argues that assimilation of voices involves incorporation of others' conceptual horizons into one's own perspective (but of course selectively), thereby incorporating the evaluative tones that individuals rework and accentuate (Bakhtin, 1986). Therefore, voices with particular utterances involve particular value judgments. The notion of value is used here in a broader sense including moral, spiritual, and worldly elements (Skulason, 1988 *cf.* Gudmundsdottir, 1990).

Returning back to Banu's discourse and practice, one can easily identify instances of her value judgments in relation to the voices existent in her speech. For example, having shown the applicability of the formal definition to obtain the derivative of a function at a point, she exclaimed that there would be short ways to find the derivative at a point (referring to rules of differentiation). On this occasion, she was in fact interacting with a past student (by responding) who was keen to learn a short way. Banu's consideration of finding the derivative through the formal definition as a "tussle" and noting that the short way in fact contains a message that emphasizes the supremacy of the

latter to the former and reflects her obvious preference for the latter. In this sense, as Gudmundsdottir (1990) argues, values always imply a choice and preference.

One can see further value judgments in Banu's discourse, such as providing examples of the real-life use of derivative as a necessity and hence doing so; attaching import to constructing the definition of a derivative via physical and geometrical interpretations, rather than supplying it directly. These value judgments reflect Banu's points of view, which come into contact with many others during different historical periods of her life. The voices and particular value judgments give a direction to the pedagogical practices of teachers, as has been evidenced in Banu's case.

Value judgments are related to what one thinks as right or wrong, good or bad, important or ignorable and these constantly guide one's practice whether consciously or not (Carbone, 1987). This argument can easily be corroborated by recourse to Banu's pedagogical practice. As become clear during the interviews, in Banu's view, some of the elements for successful teaching of mathematics are visualization, concretization, building upon existing knowledge, progressing from the simple to the complex, and constructing definitions instead of supplying them directly. For Banu, these were the right elements and important for effective mathematics instruction. She took those elements into consideration during her instruction. Hence, her values deeply impacted her choices as well as the use of pedagogical strategies in her micro-teaching lesson.

The interview excerpts provide evidence that these values were learned through dialogic interaction with the others. For example, Banu learned the importance of real-life examples through interaction with the voices of students (who questioned the usefulness of learning mathematics) and her brother (who asked about derivative while doing sketches in Autocad); and the importance of concretization through the interaction with the voices of teachers (who taught derivative in abstract ways with signs and symbols) of whom Banu was critical. These considerations suggest that PCK cannot be divorced from teachers' value judgments, which develop through the unique individual experiences of dialogic interaction with the voices of others. Value judgments about the human nature, the (worthiness of) knowledge and its production, and the direction in which we should desire knowledge are all at the heart of pedagogy. Thus, values are the cement through which content and pedagogy are brought together to create PCK. These judgments are dependent upon the unique personal experiences of individuals gained in diverse cultural sites, from various textual sources at different historical periods, pointing to the subjectivity of PCK and the roles of values in shaping this subjectivity.

6. Conclusions and implications

This paper stemmed from our attempts to better understand the subjective nature of PCK; and in this paper, we tried to make explicit the impact of others' voices and values in shaping teachers' pedagogical practices and hence the subjectivity of PCK. As has become evident with empirical data, practices and discourses of inexperienced pre-service teachers, such as those of Banu's, are imbued with others' voices and specific value judgments. Their efforts in making the subject comprehensible to learners are greatly shaped by those voices and values.

One implication of our observations and in-depth analysis provided in this paper is that PCK is a complex notion and includes subtle dynamics in its development. Literature is abundant with research studies which report on the results of PCK development programs on teachers' instructional practices (e.g. Cueto, León, Sorto, & Miranda, 2017; Şahin, Gökkurt, & Soylu, 2016). The researchers often provide participants with the knowledge of student difficulties and useful representations. These two are part of Shulman's initial conceptualization of PCK and teachers' knowledge growths in those areas are, we believe, certainly valuable. There are also others (see Park & Oliver, 2008 for a review) who add new components to PCK (e.g. knowledge of curriculum and assessment) and who tend to see teachers' PCK growth in terms of knowledge development on added components. We believe these efforts contribute to the teacher development (though not necessarily at a level intended by researchers) and hence should continue.

However, our consideration in this paper suggests there is more to the PCK development than teachers' knowledge growth. We believe it would be simplistic to focus only on teacher knowledge growth on certain components and expect a real change in teachers' PCK. We agree with Shulman (2015) when he criticizes knowledge-based definition of PCK which he terms as "pedagogical *mind*". This view sees "teachers as thinkers, as problem solvers, as decision makers ... as if they were sitting in some ivory tower" (p. 10). Therefore, Shulman suggests focusing on "pedagogical *action*" which reflects on the question of "what am I going to do with all this thinking I am doing?" (p. 10).

As this paper suggests, development of PCK takes place under subtly acting forces. Hence, any approach neglecting the power of those forces on one's PCK development would face serious difficulties to achieve real changes. Important questions arise here: To what extent do we, teacher educators, need to work with/against the voices and values appropriated by pre-service teachers? How and what kind of opportunities do we create for pre-service teachers to become aware of the effect (and thus possibly question the merit) of voices and values in their own practice? How can we get (preservice) teachers critically reflect on the effect of voices and values in their instructional practices? These, we think, are not easy questions but worth paying serious attention.

The findings of this study have also important implications for future studies on PCK. Shulman (2015) proposes a set of weaknesses and limitations of his original formulation of PCK that he introduced in earlier studies. First limitation that Shulman mentions is related to ignorance of non-cognitive attributes such as emotion, affect, feelings, moral judgments, identity and motivation. Second limitation Shulman (2015) put forward is that his original formulation of PCK insufficiently addresses social and cultural context. With regard to this, Shulman (2015) defines "pedagogical culture knowledge" and "pedagogical context knowledge" and invites researchers to consider these aspects in PCK studies. We believe that our investigation of subjectivity of PCK considering the role of voice and value partly attends to these issues and contributes to the understanding of cultural and contextual dimensions that Shulman proposes.

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